Technical Paper by N. Touze-Foltz, R.K. Rowe and C. Duquennoi

LIQUID FLOW THROUGH COMPOSITE LINERS DUE TO GEOMEMBRANE DEFECTS: ANALYTICAL SOLUTIONS FOR AXI-SYMMETRIC AND TWO-DIMENSIONAL PROBLEMS

ABSTRACT: A general framework for calculating the rate of liquid flow through a composite liner (geomembrane plus soil liner) with holes is presented. Solutions given for a circular hole and a damaged wrinkle can be used for interpreting data from laboratory tests, modelling expected field conditions, and interpreting field leakage data. A number of existing solutions arise from the general solution as special cases. Finally, the paper is extended to consider the potential interaction between damaged wrinkles, and it is theoretically shown that while this may be an important issue for composite liners incorporating a compacted clay liner, it is far less likely to be significant for those incorporating a geosynthetic clay liner.

KEYWORDS: Geomembrane, Leakage, Defect, Wrinkle, Wave.

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1 INTRODUCTION

Geomembranes used for lining municipal solid waste landfills often have holes caused by inadequate seaming, punctures, tears, etc. A recent synthesis of studies involving electrical leak detection systems (Rollin and Jacquelin 2000) reports a hole density varying from 2 to 26 defects per hectare after installation of the geomembrane. These defects form preferential advective leachate flow paths through the geomembrane. Several investigators report that liquid flowing through geomembrane defects can spread laterally between the geomembrane and the soil before infiltrating into the subsoil (Fukuoka 1986; Brown et al. 1987; Touze-Foltz 1999). A number of analytical solutions (Jayawickrama et al. 1988; Rowe 1998) have been developed to quantify liquid flow for the case of a circular hole in a flat geomembrane where there is a gap of uniform thickness between the geomembrane and the soil; this is called the “axi-symmetric case” in the current paper. Based on Brown et al. (1987), Giroud and Bonaparte (1989) and, more recently, Giroud et al. (1998) have developed semi-empirical equations for a range of cases. Analytical solutions have also been developed in the case of a damaged geomembrane wrinkle for a number of specific boundary conditions (Rowe 1998); this is called the “two-dimensional case” in the current paper.

The objective of the current paper is to propose a general framework for solving the problem of liquid flow into a composite liner (geomembrane plus soil liner) that can be reduced to the existing solutions as special cases. These solutions may be useful for interpreting experimental data from laboratory tests designed to simulate field conditions, modelling field conditions for different design scenarios, and interpreting the results from field leakage data. Consideration is given to liquid flow through composite liners for a range of boundary conditions. The governing differential equation is solved for the case of a circular hole and then for the case of a damaged wrinkle. An extension of the solution obtained for a damaged wrinkle to the resolution of the problem of liquid flow for two (or more) parallel interacting damaged wrinkles is then presented. Finally, the current paper illustrates the reduction in the rate of liquid flow that can occur due to interaction between wrinkles. For the cases examined, it will be shown that interacting wrinkles may be an important issue for composite liners that include a compacted clay liner, but is less likely to be significant for composite liners that include a geosynthetic clay liner.

2 ASSUMPTIONS AND LIMITATIONS

The assumptions and limitations described below refer to the geometry and hydraulics related to solving the problems of liquid flow through a composite liner due to: (i) a circular hole in a flat geomembrane forming part of a composite liner; and (ii) a damaged wrinkle in a geomembrane. Common assumptions to both cases are presented in Section 2.1. Specific assumptions are detailed in Sections 2.2 and 2.3 for the axi-symmetric (circular hole in flat surface) and two-dimensional (hole in a wrinkle) cases, respectively.

2.1 Common Assumptions

The general liner system considered (Figure 1) follows from Rowe (1998) and includes a geomembrane resting on a low-permeability clay liner of thickness H_L and hydraulic conductivity k_L. This low-permeability clay liner can be either a compacted clay liner (CCL) or a geosynthetic clay liner (GCL) and will be simply called a “soil liner”. The z-axis origin corresponds to the top of the soil liner with upward being positive. The soil liner rests on a more permeable foundation or attenuation layer, of thickness H_f and hydraulic conductivity k_f, which itself rests on a highly permeable layer that can be either an aquifer or a leakage collection layer. Following from Brown et al. (1987) and Giroud and Bonaparte (1989), it is assumed that the geomembrane is not in perfect contact with the soil liner and that there is a uniform transmissive zone between the geomembrane and the soil liner surface that will be referred to as the “transmissive layer”. The hydraulic transmissivity, θ, of this transmissive layer is established based on experimental data. An example of an experimental device used to measure hydraulic transmissivity is presented in Section 5.

It is assumed that: (i) liquid flow is under steady-state conditions; (ii) the soil liner and the foundation layer are saturated; and (iii) liquid flow through the liner and foundation layer is vertical. Thus, based on continuity of liquid flow, the equivalent hydraulic conductivity, k_s, corresponding to the liner and the foundation layer is given by:

$$\frac{H_L + H_f}{k_s} = \frac{H_L}{k_L} + \frac{H_f}{k_f} \tag{1}$$

When a hydraulic head, h_w, is applied on top of the composite liner, the maximum mean hydraulic gradient, i_s, through the liner and foundation is given by:

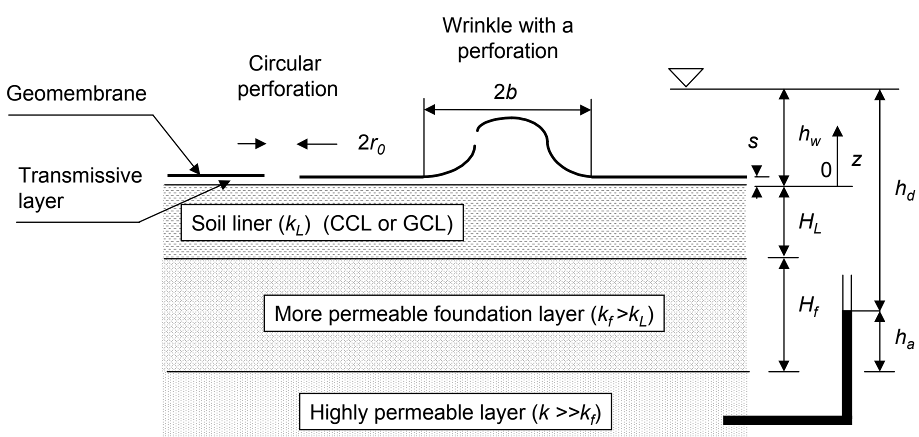

Figure 1. Schematic showing a hole of radius r_0 and a wrinkle with a perforation in a geomembrane and the underlying strata (modified from Rowe(1998)).

$$i_s = 1 + \frac{h_w - h_a}{H_L + H_f} \tag{2}$$

where h_a is the hydraulic head in the highly permeable layer.

2.2 Specific Assumptions for the Axi-Symmetric Case

Assuming a uniform hydraulic transmissivity and a circular hole of radius r_0 , the liquid flow in the transmissive layer is radial, and the problem is axi-symmetric. The system considered is a cylinder of radius R_c with a central hole of radius r_0 in the geomembrane (Figures 2a and 2b). R_c can be either the physical radius of a cell in the case of a laboratory test or, for field conditions, a virtual radius as discussed in Section 3.1.4. This cylinder contains, from top to bottom, all of the layers presented in Figure 1.

Assuming the soil is saturated, the boundary condition in the transmissive layer at $r = R_c$ is either:

- zero flow (Figure 2a), namely:

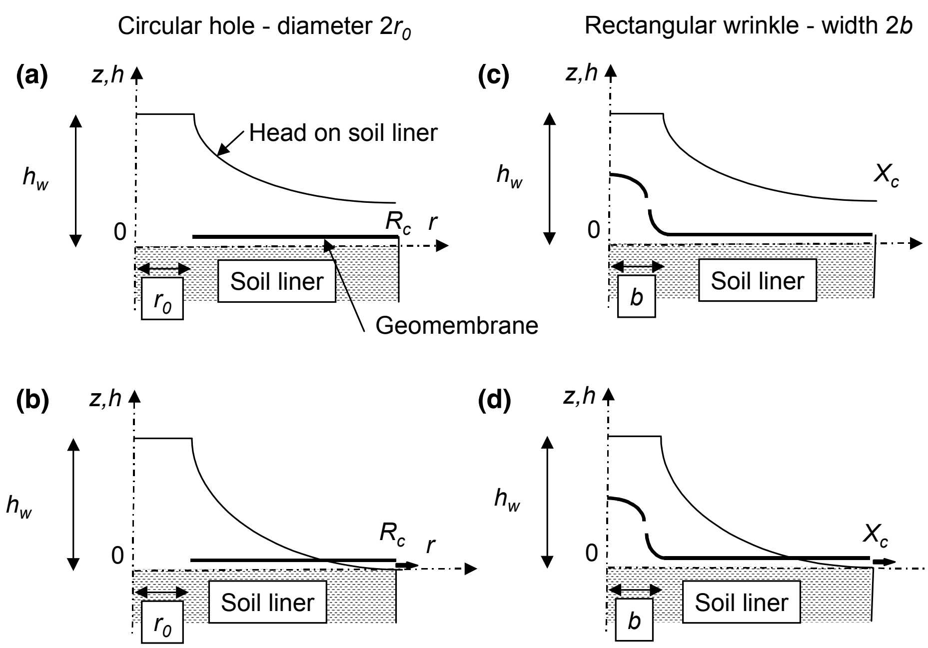

Figure 2. Hydraulic head on the soil liner: (a) no flow boundary condition at $r = R_c$; (b) specified head ($h_s = 0$) boundary condition at $r = R_c$; (c) zero flow boundary condition at $x = X_c$; (d) specified head ($h_s = 0$) boundary condition at $x = X_c$.

$$Q_r(R_c) = 0 \tag{3a}$$

and, hence, in general

$$h(R_c) \geq 0 \tag{3b}$$

- or, a specified head, namely:

$$h(R_c) = h_s \tag{4a}$$

and, hence, in general

$$Q_r(R_c) \geq 0 \tag{4b}$$

where: Q_r = radial rate of liquid flow in the transmissive layer; h_s = specified hydraulic head in the transmissive layer at $r = R_c$ (where r is the radial boundary) ($h_s = 0$ in Figure 2b); and h = hydraulic head in the transmissive layer.

2.3 Specific Assumptions for the Case of a Hole in a Wrinkle

One can consider the case of a damaged rectilinear wrinkle of length L and width $2b$, with $L>>b$ so that the effects of liquid flow at the ends of the wrinkle can be neglected (Figure 1). No particular assumptions are made regarding the dimension, position, or the number of holes in the wrinkle, but rather it is assumed that the rate of liquid flow in the composite liner is not limited by the holes (the hole-limiting case is discussed by Rowe (1998)). This will be discussed further in Section 5. Liquid flow in the transmissive layer is assumed to be in the x-direction (Figure 2), normal to the longitudinal axis of the wrinkle. Under the assumption of a uniform hydraulic transmissivity, the problem of liquid flow becomes two-dimensional. The system considered is then a parallelepiped of width $2X_c$, with a central wrinkle as shown in Figures 2c and 2d, where $2X_c$ is the physical width of a cell in the laboratory or, for field conditions, a virtual width that can possibly tend toward infinity. This parallelepiped contains, from top to bottom, all of the layers shown in Figure 1.

Boundary conditions considered at $x = X_c$ are:

- zero flow (Figure 2c):

$$Q_x(X_c) = 0 \tag{5a}$$

$$h(X_c) \geq 0 \tag{5b}$$

- or, a specified head:

$$h(X_c) \geq h_s \tag{6a}$$

$$Q_x(X_c) \geq 0 \tag{6b}$$

where: Q_x = rate of liquid flow in the transmissive layer in the direction normal to the longitudinal axis of the wrinkle; h_s = specified head at X_c ($h_s = 0$ in Figure 2d); and x = horizontal boundary. The solution to be developed in Section 3 assumes known hydraulic conductivities k_L and k_f for saturated conditions. For saturated conditions to be maintained, $h_s \geq 0$ in Equations 4a and 6a. However, it is possible for suction to develop in the soil as it begins to desaturate and that, remote from the wrinkle, the minimum value of $h_s = h_a - (H_L + H_f)$. The hydraulic conductivities become a function of soil suction for x such that $h(x) \leq 0$, and, because the solution is not strictly correct in this case, any application in this domain will require engineering judgement. In terms of calculating rates of liquid flow, the assumption that the solution is valid for $h_s \geq h_a - (H_L + H_f)$ is likely to be conservative; however, more research is required to confirm this hypothesis.

The damaged wrinkle need not necessarily be symmetrical with respect to its shape or the location of the holes provided that the gap beneath the wrinkle is large compared to the thickness of the transmissive layer between the geomembrane and the soil liner. Under these circumstances, there will be negligible head loss beneath the wrinkle and it is assumed that all of the head loss at the interface occurs between the geomembrane and the soil liner outside the wrinkle.

3 HYDRAULIC HEAD PROFILE BELOW A GEOMEMBRANE AND RATE OF LIQUID FLOW THROUGH COMPOSITE LINERS

The calculation of the rate of liquid flow through the composite liner is linked to the hydraulic head profile that can be obtained by solving the mass conservation equation (Brown et al. 1987). In the following, the axi-symmetric and two-dimensional cases are treated separately because the governing differential equations obtained in both cases are different.

3.1 Solution for the Axi-Symmetric Case

3.1.1 *Governing Equation*

The radial rate of liquid flow in the transmissive layer at the distance r from the circular-hole axis, $Q_r(r)$, can be expressed as (Brown et al. 1987):

$$Q_r(r) = -2 \pi r \theta \frac{\mathrm{d}h}{\mathrm{d}r} \tag{7}$$

Using Darcy's Law and assuming the soil is saturated, the rate of liquid flow, $\mathrm{d}Q_s(r)$, infiltrating into the ring of soil (soil liner and the foundation layer) between radii r and $r + \mathrm{d}r$ is:

$$\mathrm{d}Q_s(r) = -2 \pi r k_s \left(1 + \frac{h - h_a}{H_f + H_L}\right) \mathrm{d}r \tag{8}$$

The mass conservation equation allows for the rate of liquid flow entering the hole to be subdivided between the rate of liquid flow infiltrating into the soil liner and the rate of liquid flow spreading laterally in the transmissive layer (Brown et al. 1987). Differentiating the mass conservation equation (Brown et al. 1987), one can obtain:

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} - \alpha^2 h = \alpha^2 C \qquad (9)$$

where the hydraulic head, h, in the transmissive layer is unknown and α and C are given by (Rowe 1998):

$$\alpha = \sqrt{\frac{k_s}{(H_L + H_f)\,\theta}} \qquad (10)$$

$$C = H_L + H_f - h_a \qquad (11)$$

3.1.2 *General Solution*

Brown et al. (1987) gave a general solution of Equation 9 that can be written in the form:

$$h(r) = AI_0(\alpha r) + BK_0(\alpha r) - C \qquad \text{for } r_0 \le r \qquad (12)$$

where: K_0 and I_0 = modified Bessel functions of zero order; and A and B = constants. Equation 9 can now be solved for one of the boundary conditions at $r = R_c$ as proposed in Equations 3 and 4. The two constants, A and B, must be evaluated, thus, a second boundary condition is required. This condition corresponds to the head at the hole in the geomembrane, h_w :

$$h(r_0) = h_w \qquad (13)$$

3.1.3 *Solution for Zero Flow $Q_r = 0$ at $r = R_c$*

When solving Equation 12 for the boundary conditions defined by Equations 3a and 13, one can obtain the following, a variant of the solution given by Brown et al. (1987):

$$h(r) = A_Q I_0(\alpha r) + B_Q K_0(\alpha r) - C \qquad (14)$$

where:

$$A_Q = \frac{(h_w + C)\, K_1\,(\alpha R_c)}{K_1\,(\alpha R_c)\, I_0\,(\alpha r_0) + K_0\,(\alpha r_0)\, I_1\,(\alpha R_c)} \tag{15}$$

$$B_Q = \frac{(h_w + C)\, I_1\,(\alpha R_c)}{K_1\,(\alpha R_c)\, I_0\,(\alpha r_0) + K_0\,(\alpha r_0)\, I_1\,(\alpha R_c)} \tag{16}$$

Equation 14 is valid for any r such that $r_o \leq r \leq R_c$ provided that $h(R_c) \geq 0$. K_1 and I_1 are modified Bessel functions of first order. Based on the consideration of continuity of liquid flow, the total rate of liquid flow, Q, in the composite liner is equal to the sum of the rate of liquid flow into the soil liner below the hole ($r \leq r_o$) and outside the hole ($r_o < r \leq R_c$) and is given by:

$$Q = \pi\, r_0^2\, k_s\, i_s - 2\pi\, r_0\, \theta\, \alpha\, [A_Q\, I_1\,(\alpha r_0) - B_Q\, K_1\,(\alpha r_0)] \tag{17}$$

3.1.4 *Solution for Specific Head $h = h_s$ at $r = R_c$*

Solving Equation 12 for the boundary conditions defined by Equations 4a and 13 gives:

$$h(r) = A_p\, I_0\,(\alpha r) + B_p\, K_0\,(\alpha r) - C \tag{18}$$

for $r_o \leq r \leq R_c$ and $Q_r(R_c) \geq 0$, where

$$A_p = -\frac{(h_w + C)\, K_0\,(\alpha R_c) - (h_s + C)\, K_0\,(\alpha r_0)}{K_0\,(\alpha r_0)\, I_0\,(\alpha R_c) - K_0\,(\alpha R_c)\, I_0\,(\alpha r_0)} \tag{19a}$$

$$B_p = \frac{(h_w + C)\, I_0\,(\alpha R_c) - (h_s + C)\, I_0\,(\alpha r_0)}{K_0\,(\alpha r_0)\, I_0\,(\alpha R_c) - K_0\,(\alpha R_c)\, I_0\,(\alpha r_0)} \tag{19b}$$

which, for the special case of $h_s = 0$ (Figure 2b), reduces to:

$$A_p = -\frac{h_w\, K_0\,(\alpha R_c) + C\,[K_0\,(\alpha R_c) - K_0\,(\alpha r_0)]}{K_0\,(\alpha r_0)\, I_0\,(\alpha R_c) - K_0\,(\alpha R_c)\, I_0\,(\alpha r_0)} \tag{20a}$$

$$B_p = \frac{h_w\, I_0\,(\alpha R_c) + C\,[I_0\,(\alpha R_c) - I_0\,(\alpha r_0)]}{K_0\,(\alpha r_0)\, I_0\,(\alpha R_c) - K_0\,(\alpha R_c)\, I_0\,(\alpha r_0)} \tag{20b}$$

Rowe (1998) has solved Equation 9 for the particular case where:

$$\begin{cases} Q_r\,(R_c) = 0 \\ h(R_c) = 0 \end{cases} \tag{21}$$

This case can be interpreted as the limiting case where the solutions obtained for both types of boundary conditions, zero flow and zero head at $r = R_c$, are equivalent. The radius $R_c = R_w$ for which Equation 21 is satisfied is known as the radius of the wetted area and represents the limit of validity of solutions to Equations 14 and 18. It is possible to find solutions to Equations 14 and 18 for $R_c > R_w$; however, for radii greater than R_w , the soil is no longer fully saturated, and the solutions thus obtained by Equations 14 and 18 are only approximate solutions.

The total rate of liquid flow, Q , infiltrating the hole is obtained in the same way as for the previous boundary conditions. However, the rate of liquid flow, Q , is greater than the rate of liquid flow infiltrating into the soil liner, Q_s , because $Q_r(R_c) > 0$ except when $R_c = R_w$. Expressions for the rates of liquid flow Q, $Q_r(R_c)$, and Q_s are given by Equations 22 to 24, respectively:

$$Q = \pi r_0^2 k_s i_s - 2\pi r_0 \theta \alpha [A_p I_1(\alpha r_0) - B_p K_1(\alpha r_0)] \tag{22}$$

$$Q_r(R_c) = -2\pi \theta \alpha R_c [A_p I_1(\alpha R_c) - B_p K_1(\alpha R_c)] \tag{23}$$

$$Q_s = \pi r_0^2 k_s i_s - 2\pi \theta \alpha \left\{ r_0 [A_p I_1(\alpha r_0) - B_p K_1(\alpha r_0)] - R_c [A_p I_1(\alpha R_c) - B_p K_1(\alpha R_c)] \right\} \tag{24}$$

This solution is of great interest for interpreting hydraulic transmissivity interface measurements such as those described by Fukuoka (1986) or Harpur et al. (1994). Experiments can be interpreted taking into account the liquid flow into the soil liner, such that the evaluated hydraulic transmissivity is no longer an apparent hydraulic transmissivity but the real hydraulic transmissivity. This interpretation can be made only for $Q_r(R_c) > 0$.

3.2 Solution for the Two-Dimensional Case

3.2.1 *Governing Equation*

The horizontal rate of liquid flow in the transmissive layer at a distance x from the middle of the wrinkle, $Q_x(x)$, can be expressed, by analogy with the axi-symmetric case, for one side of the wrinkle, as:

$$Q_x(x) = -L\theta \frac{dh}{dx} \tag{25}$$

Using Darcy's Law and assuming the soil is saturated, the rate of liquid flow, $dQ_s(x)$, infiltrating into the strip of soil between abscissas x and $x + dx$ is, for one side of the wrinkle:

$$dQ_s(x) = k_s \left(1 + \frac{h - h_a}{H_L + H_f}\right) L\, dx \tag{26}$$

Using the same principle of differentiation of the mass conservation equation as for the axi-symmetric case, Rowe (1998) obtained the following differential equation for the hydraulic head in the transmissive layer:

$$\frac{d^2h}{dx^2} - \alpha^2 h = \alpha^2 C \tag{27}$$

where α and C are given by Equations 10 and 11, respectively.

3.2.2 *General Solution*

The general solution of Equation 27 can be written as:

$$h(x) = E\ \exp(-\alpha x) + F\ \exp(\alpha x) - C \qquad \text{for} \quad b \leq x \tag{28}$$

where E and F are coefficients with values that depend on boundary conditions. As for the circular case, two boundary conditions are needed to solve Equation 27 and evaluate the coefficients E and F. Once again the fact that the hydraulic head of the liquid entering the hole in the geomembrane is equal to h_w provides one boundary condition:

$$h(b) = h_w \tag{29}$$

The other boundary condition is given by either Equation 5 or 6, and the solutions for these two cases are given in Sections 3.2.3 and 3.2.4, respectively.

3.2.3 *Solution for Zero Flow at $x = X_c$*

Solving Equation 27 subject to boundary conditions given by Equations 5 and 29, one obtains the following expression for the hydraulic head below the geomembrane:

$$h(x) = \frac{(h_w + C)\ \cosh[\alpha(X_c - x)]}{\cosh[\alpha(X_c - b)]} - C \tag{30}$$

for $b \leq x \leq X_c$ subject to the condition $h(X_c) \geq 0$.

Still using the principle of mass conservation, the total rate of liquid flow in the composite liner is equal to the rate of liquid flow into the soil liner and, assuming that the transmissive layer is identical on both sides of the wrinkle and has the same boundary condition at $x = X_c$, is given by:

$$Q = 2\,L\,k_s\,i_s \left(b + \frac{1}{\alpha}\ \tanh[\alpha(X_c - b)] \right) \tag{31}$$

3.2.4 *Solution for Specified Head $h = h_s$ at $x = X_c$*

Solving Equation 28 subject to the boundary conditions given by Equations 6 and 29 gives:

$$h(x) = \frac{(h_w + C)\ \sinh[\alpha(X_c - x)] - (h_s + C)\ \sinh[\alpha(b - x)]}{\sinh[\alpha(X_c - b)]} - C \qquad (32)$$

for $b \leq x \leq X_c$ provided that $Q_x(X_c) \geq 0$.

The total rate of liquid flow Q infiltrating the hole is obtained in the same way as for the axi-symmetric case. Expressions of Q, Q_s , and $Q_x(X_c)$ are given by Equations 33 to 35, respectively:

$$Q = 2Lk_s \left\{ bi_s + \frac{(h_w + C)\ \cosh[\alpha(X_c - b)] - (h_s + C)}{\alpha(H_L + H_f)\ \sinh[\alpha(X_c - b)]} \right\} \qquad (33)$$

$$Q_s = 2Lk_s \left\{ bi_s + \frac{(h_w + h_s + 2C)\ \{\cosh[\alpha(X_c - b)] - 1\}}{\alpha(H_L + H_f)\ \sinh[\alpha(X_c - b)]} \right\} \qquad (34)$$

$$Q_x\,(X_c) = 2Lk_s \left\{ \frac{h_w + C - (h_s + C)\ \cosh[\alpha(X_c - b)]}{\alpha(H_L + H_f)\ \sinh[\alpha(X_c - b)]} \right\} \qquad (35)$$

Equations 30 and 32 provide the hydraulic head profile below the geomembrane and Equations 31 and 33 to 35 provide the expressions for rates of liquid flow.

For the two-dimensional case, the only hypotheses made regarding the wrinkles in obtaining Equations 30 and 32 are that the holes are not limiting the rate of liquid flow and that the width of the wrinkle is equal to 2*b*. This means that, provided holes do not limit the rate of liquid flow through the composite liner, the situation is exactly equivalent to that of a rectangular defect of width 2*b* and length *L*, with $L >> b$, as no assumption is made regarding the height of the wrinkle nor its shape.

The solution obtained in the case of a zero flow boundary condition at $x = X_c$ is given by Equations 30 and 31 and is of particular interest in the case of two interacting parallel wrinkles (i.e. when the head below the geomembrane between the two wrinkles is strictly positive). A solution for this case is given in Section 4.

3.2.5 *Particular Cases of Interest*

A particular case that can be considered is the one in which the hydraulic head is equal to zero at boundary X_c (i.e. $h_s = 0$). As for the circular case, this particular value of $X_c = X_w$ gives the limit of strict validity of both solutions obtained for zero flow and specified head boundary conditions given by Equations 30 and 32 respectively, where:

$$X_w = \frac{1}{\alpha} \cosh^{-1}\left(\frac{h_w + C}{C}\right) + b \qquad (36)$$

In this case, the expression of the hydraulic head becomes:

$$h(x) = 2C \sinh^2\left[\alpha \frac{(X_w - x)}{2}\right] \quad (37)$$

Values of x greater than X_w are possible, but for this case $h(x)$ is negative and the soil begins to desaturate as suction develops below the geomembrane. In the limit $h(x) \to -C$ and as shown by Rowe (1998) for $h(X_c) = -C$:

$$\cosh\left[\alpha(X_c - b)\right] \to +\infty \quad (38)$$

and, thus

$$X_c \to +\infty \quad (39)$$

Therefore, the value of X_c that gives $Q_x(X_c) = 0$ is greater than the limit value of X_c (i.e. X_w) obtained from Equation 36. It follows that the solution in this case is an approximate solution because saturated and unsaturated zones are present in the composite liner.

4 RATE OF LIQUID FLOW REDUCTION FOR TWO INTERACTING WRINKLES

4.1 Overview on Geomembrane Wrinkles

In the field, geomembranes expand when they are heated by the sun and wrinkles (sometimes called waves) appear. The size and spacing of these wrinkles have been shown to depend on temperature and on geomembrane characteristics such as colour, coefficient of thermal expansion, roughness, and flexibility. Giroud and Morel (1992) theoretically calculated the distance between two adjacent parallel wrinkles of infinite length, l_{bw} , to be of the order of 10 m with a wrinkle height of the order of 0.1 m for high density polyethylene (HDPE). For a polyvinyl chloride (PVC) geomembrane, l_{bw} was calculated by Giroud and Morel (1992) to be of the order of 1 m with a wrinkle height of the order of 0.01 m. Pelte et al. (1994) compared site measurements and mathematical calculations and found that, for a 1.5 mm-thick HDPE geomembrane, l_{bw} was of the order of 5 m with a wrinkle height of approximately 0.1 m and a wrinkle half-width, b, of approximately 0.15 m.

4.2 Problem Definition

The problem is shown schematically in Figure 3 and involves two parallel wrinkles that are long enough to neglect the end effects. It is assumed that both the hydraulic head on top of the geomembrane and the hydraulic transmissivity between the geomembrane and the soil liner are uniform. Between the two wrinkles, the hydraulic head reaches a minimum, which corresponds to a zero horizontal flow condition below the geomembrane, according to Equation 25. As a consequence, the problem can be treated as shown in Figure 3b, by decoupling the solution for Wrinkles 1 and 2, but enforcing a zero flow boundary condition between the two wrinkles.

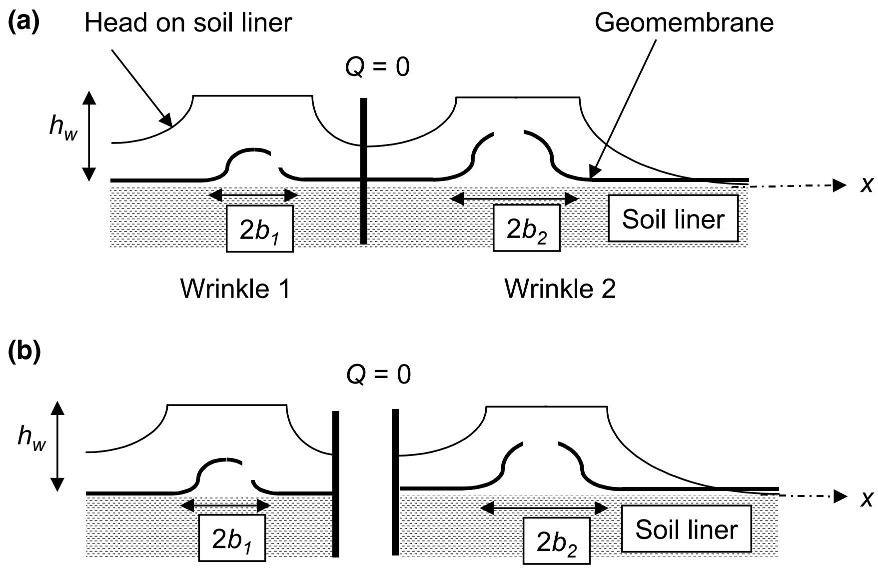

Figure 3. Schematic showing: (a) the way two geomembrane wrinkles with holes interact with the no-flow-plane in between them; (b) hydraulic decoupling used to obtain the analytic solution.

Resolving the problem of liquid flow for two interacting wrinkles is equivalent to the resolution of four simple problems, for half wrinkles, for which the solutions are known: two have zero flow boundary conditions between the two wrinkles and, for the other two, the boundary condition can be either a zero flow condition, in the case of interaction with another parallel wrinkle, or the presence of a physical boundary, or a specified head boundary condition.

The only difficulty is that the position of the zero flow plane between both wrinkles is not known a priori, but can be deduced as indicated in Section 4.3.

4.3 Head Profile and Rate of Liquid Flow for Two Interacting Damaged Wrinkles

Figure 4 shows the system examined and the notation used. A subscript 1 refers to the wrinkle on the left-hand side of Figure 4 and subscript 2 to the wrinkle on the right-hand side. The origin of abscissa is positioned at the centre of Wrinkle 1. The length between wrinkles, l_{bw} , is the total distance between the centres of Wrinkles 1 and 2:

$$l_{bw} = l_1 + l_2 \tag{40}$$

where l_1 and l_2 are the width of Wrinkles 1 and 2 in the case of two interacting wrinkles, respectively.

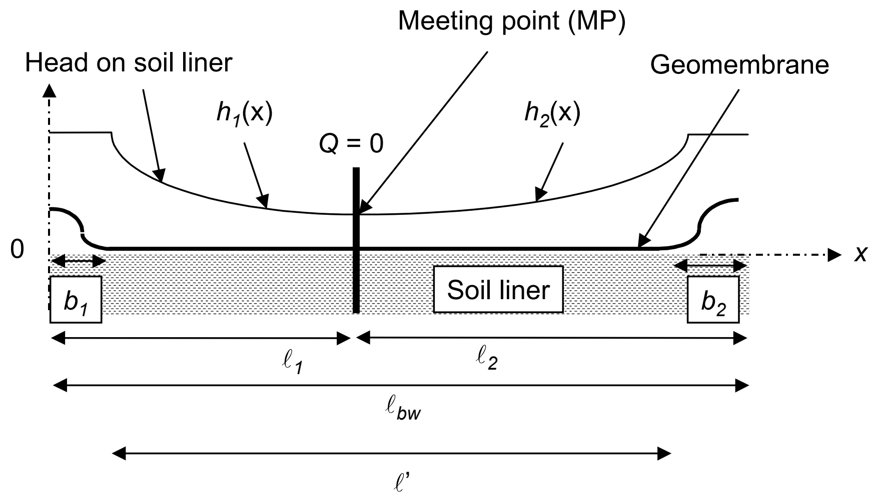

Figure 4. Schematic showing the zone of interaction between two geomembrane wrinkles and the notation used.

Based on Equation 30, the hydraulic head h_1 for x between b_1 and l_1 (where b_1 and b_2 are the half-widths of Wrinkles 1 and 2 for two interacting wrinkles, respectively), and the hydraulic head, h_2 , between l_1 and l_{bw} - b_2 are given by Equations 41 and 42, respectively:

$$h_1(x) = \frac{(h_w + C)\ \cosh[\alpha(l_1 - x)]}{\cosh[\alpha(l_1 - b_1)]} - C \tag{41}$$

$$h_2(x) = \frac{(h_w + C)\ \cosh[\alpha(l_1 - x)]}{\cosh[\alpha(l_2 - b_2)]} - C \tag{42}$$

Because the hydraulic head is a continuous quantity, the abscissa of the zero flow plane must satisfy:

$$h_1\,(l_1) = h_2\,(l_1) \tag{43}$$

and, hence, from Equations 41 to 43:

$$l_1 - b_1 = l_2 - b_2 \tag{44}$$

Combining Equations 40 and 44, one can deduce the coordinates of the meeting point, MP, of the two hydraulic head profiles, namely:

$$(x,\ h) = \left[\frac{l_{bw} + b_1 - b_2}{2}\ ;\ \frac{(h_w + C)}{\cosh\left(\alpha\ \frac{l_{bw} - b_1 - b_2}{2}\right)} - C\right] \tag{45}$$

It follows from Equation 45 that MP is the midpoint between the edges of Wrinkles 1 and 2. This should not be a surprise because all hydraulic parameters considered herein are uniform. This results in a symmetrical expression for the hydraulic head profile below the geomembrane that can be obtained by combining the two expressions of hydraulic head given by Equations 41 and 42, namely:

$$h(x) = \frac{(h_w + C)\ \cosh\left[\alpha\left(\frac{l_{bw} + b_1 - b_2}{2} - x\right)\right]}{\cosh\left[\alpha\left(\frac{l_{bw} - b_1 - b_2}{2}\right)\right]} - C \tag{46}$$

Using Equation 31 to calculate the rate of liquid flow for Wrinkles 1 and 2 separately, one can obtain the total rate of liquid flow between the wrinkles, taking into account the liquid flow into the soil liner under the wrinkles:

$$Q = Lk_s\, i_s \left\{b_1 + b_2 + \frac{2}{\alpha}\ \tanh\left[\alpha\left(\frac{l'}{2}\right)\right]\right\} \tag{47}$$

where:

$$l' = l_{bw} - b_1 - b_2 \tag{48}$$

The rate of liquid flow contribution, Q', from the zone between the edges of the wrinkles is given by:

$$Q' = \frac{2Lk_s\, i_s}{\alpha}\ \tanh\left[\alpha\left(\frac{l'}{2}\right)\right] \tag{49}$$

One should notice that l' and Q' are independent of the width of both wrinkles. The solution only depends on the distance between the wrinkles and their length, but not on any other physical characteristics of the wrinkles.

If there is a periodic group of wrinkles, then, Equation 47 defines the rate of liquid flow per wrinkle. If one is only considering a pair of wrinkles, then, the total rate of liquid flow is obtained by combining the rate of liquid flow given by Equation 47 with the flow calculations for the left half of Wrinkle 1 and right half of Wrinkle 2 as appropriate for the given boundary conditions.

5 EXAMPLE CALCULATIONS

5.1 Experimental Determination of Hydraulic Transmissivity

The hydraulic transmissivity of the transmissive layer is an essential parameter for the calculation of rates of liquid flow through composite liners as shown by Equations 17, 22 to 24, 31, and 33 to 35, which give rate of liquid flow in the soil and at the end of the transmissive layer, both for the axi-symmetric and the two-dimensional cases. However, there is a paucity of direct measurements of this parameter. Fukuoka (1986) and Liu (1998) have reported experimental results for loamy soils in contact with PVC geomembranes, and Harpur et al. (1994) considered GCLs in contact with an HDPE geomembrane. In all of these cases, the rates of liquid flow were interpreted without taking into account liquid flow in the soil liner. The solution obtained for the axi-symmetric and two-dimensional cases for a specified head boundary condition at the edge of the transmissive layer allows the evaluation of θ, while considering liquid flow into the soil. The cell shown in Figure 5 has been specially designed for measuring hydraulic transmissivity. In the bottom part of this cell, 60 mm of soil, which can be either loam or clay, is compacted. On top of it, a geomembrane with a circular hole in its centre is placed. In the current experiments, the diameter of the hole varies from 1 to 3 mm. The geomembrane is covered with granular materials, simulating the presence of the protecting soil layer. The vertical stress exerted by this layer is approximately 1 kPa. The bottom boundary condition is defined by $h_a = 0$.

Liquid flow measurement can be conducted in two different ways as described by Harpur et al. (1994). Constant head tests are carried out when $Q_r(R_c)$ is large. When $Q_r(R_c)$ can no longer be measured, a falling head test is conducted and then the total rate

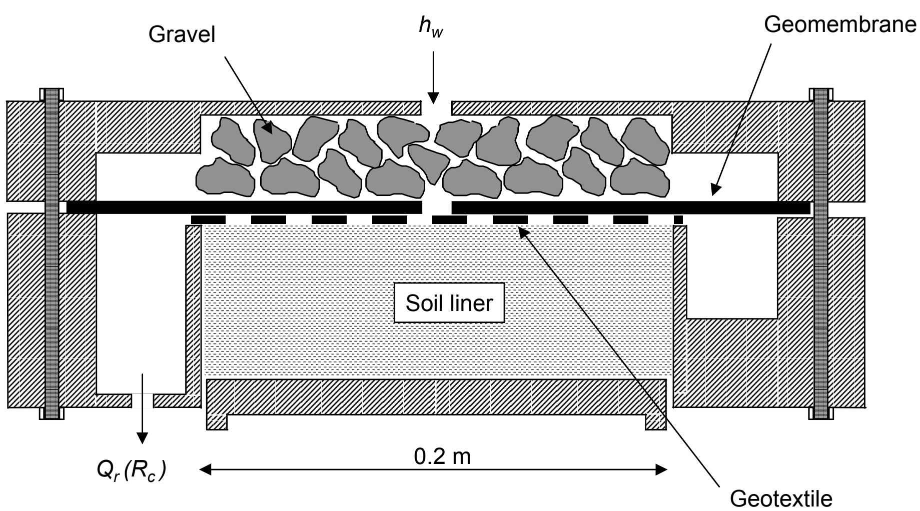

Figure 5. Schematic of the interface transmissivity measurement cell.

of liquid flow Q is measured. The results obtained from tests where $Q_r(R_c) > 0$ can be interpreted using Equation 23.

5.2 Limit of Validity of the Equations for the Two-Dimensional Case Regarding Dimension and Position of Wrinkle Damage

One of the assumptions made for addressing the problem of liquid flow through a composite liner with a damaged wrinkle is that neither the hole(s) in the wrinkle nor the waste layer or leachate collection layer on top of the geomembrane are limiting the rate of liquid flow. As indicated by Rowe (1998), this limitation could be due to the dimension of the hole or to the hydraulic conductivity of the overlying medium. In both cases, the rate of liquid flow, Q, can be obtained iteratively by the modified form of the equation given by Giroud et al. (1997) for the case of a plane geomembrane. This form is modified because the rate of liquid flow is not only in relation to the hydraulic head on top of the liner, h_w, but to the hydraulic head on top of the hole, $h_w - z_{hole}$. The notation is defined in Figure 6. One thus obtains:

$$h_w - z_{hole} = \left\{ \frac{aq_i}{2k_{OM}\,\pi} + \frac{Q}{2k_{OM}\,\pi}\left[\ln\left(\frac{Q}{aq_i}\right) - 1\right] + \frac{1}{4g^2}\left(\frac{Q}{0.6a}\right)^4 \right\}^{1/2} \quad (50)$$

where: q_i = liquid supply (i.e. vertical percolation or flow per unit plan area of a landfill) as defined by Giroud et al. (1997); k_{OM} = hydraulic conductivity of the overlying me-

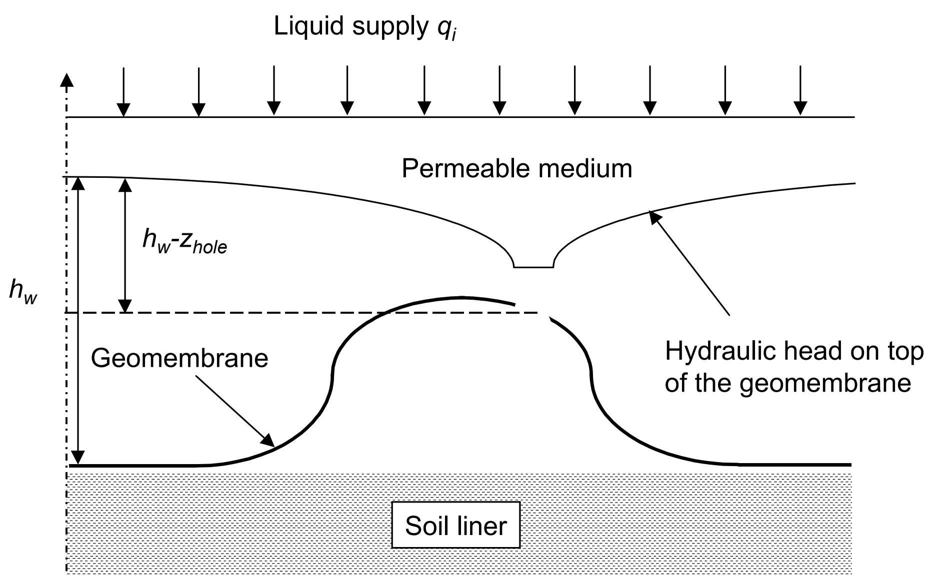

Figure 6. Definition of parameters used (modified from Giroud et al. (1997)).

dium; z_{hole} = vertical position of the center of the hole; a = area of the hole; and g = gravitational acceleration.

In order to assess whether the liquid flow is limited by the hole, the authors of the current paper have defined a nondimensional rate of liquid flow, equal to the ratio of the liquid flow given by Equation 31 for the limiting value of $X_c = X_w$ (where X_w is given by Equation 36), divided by the liquid flow given by Equation 50. Values of the ratio $Q(31)/Q(50)$ greater than one indicate that the liquid flow is limited by the hole and/or the overlying medium, whereas values of the ratio $Q(31)/Q(50)$ smaller than one indicate that the liquid flow is limited by the composite liner.

Figure 7 presents the results obtained for a strip wrinkle 0.2 m wide, 3 m long, and over a clay liner, with $k_L = 10^{-9}$ ms^{-1} and H_L = 0.6 m. The data reported are nondimensional rates of liquid flow, as a function of the height of liquid above the hole, $h_w - z_{hole}$. Calculations were performed assuming a small hole, 10^{-3} m in diameter, in the wrinkle. The rate of percolation, q_i, through the waste was varied from 10^{-9} to 10^{-7} ms^{-1}, and the hydraulic conductivity of the leachate collection system was taken to be $k_{OM} = 10^{-4}$ ms^{-1}. For a percolation rate of 10^{-7} ms^{-1}, the liquid flow will be limited by the composite liner and not by the hole nor the overlying medium, if the hydraulic head on top of the hole, $h_w - z_{hole}$, is greater than the following approximate values:

- 0.011 m for a hydraulic head equal to 0.1 m;
- 0.015 m for a hydraulic head equal to 0.3 m; and

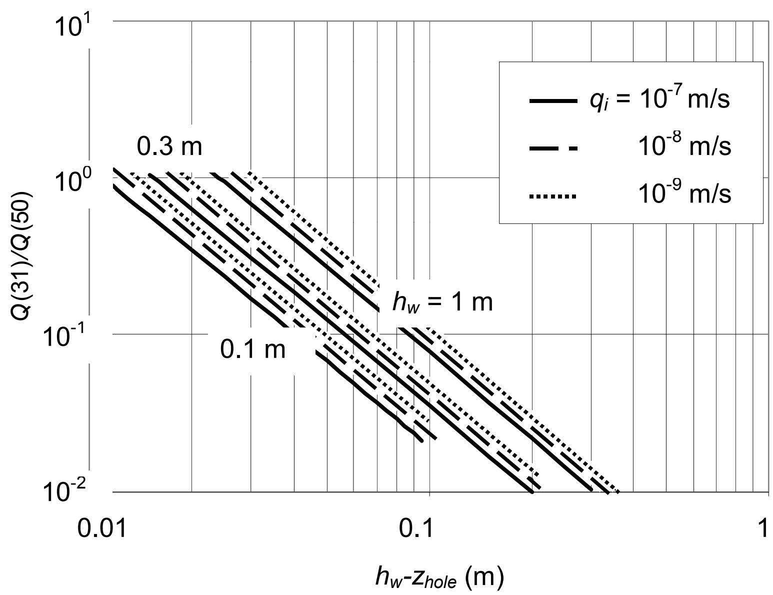

Figure 7. Evolution of the nondimensional flow rate as a function of $h_w - z_{hole}$, for different hydraulic heads on top of the geomembrane and various percolation rates through the waste, for a 3 m-long wrinkle.

- 0.024 m for a hydraulic head equal to 1 m.

In such a case, the nondimensional rate of liquid flow is less than one. Figure 7 also shows that a variation of the rate of percolation from 10^{-7} to 10^{-9} ms^{-1} results in an increase of less than 0.01 m in the minimum hydraulic head on top of the hole, $h_w - z_{hole}$, required for the liquid flow to be limited by the composite liner and not the hole; the lower the liquid supply, the higher the minimum hydraulic head required above the hole.

In the case of low hydraulic heads, the liquid flow can be limited by the hole and/or the overlying medium. For example, if one assumes the existence 0.1 m-high wrinkle, with a hole located 0.005 m vertically below the top of the wrinkle and a leachate head of 0.1 m on the composite liner, then the hydraulic head on top of the hole, $h_w - z_{hole}$ = 0.005 m. According to approximate values of limit hydraulic heads calculated previously, this corresponds to the case where the nondimensional rate of liquid flow is greater than one, and, hence, the hole (and Equation 50) controls the leakage.

5.3 Limit of Application of the Equations Obtained for Two Interacting Wrinkles

Assuming that the soil is saturated, the hydraulic head at MP, the meeting point of hydraulic head profiles between two interacting wrinkles, has a positive value. The limit of interaction of two wrinkles then corresponds to a hydraulic head equal to zero at MP. In such a case, l', defined by Equation 48, is at its maximum value. One can evaluate this maximum l' value for different composite liners. In the following calculations, the values of hydraulic conductivities of CCLs and GCLs and of hydraulic transmissivities of the transmissive layer between CCLs or GCLs and geomembranes given by Rowe (1998) are adopted. Hydraulic transmissivities of the transmissive layer between geomembranes and GCLs are in the range of 6×10^{-12} to 2×10^{-10} m^2s^{-1} based on tests performed by Harpur et al. (1994). The hydraulic conductivity of GCLs lies within a wide range of values from 7×10^{-12} to 2×10^{-8} ms^{-1} when permeated with leachate based on the published data summarised by Rowe (1998).

Two values of hydraulic transmissivity are used for the transmissive layer between geomembranes and CCLs: (i) $\theta = 1.6 \times 10^{-8}$ m^2s^{-1} when $k_L = 10^{-9}$ ms^{-1} for the CCL; and (ii) $\theta = 3.2\ 10^{-9}$ m^2s^{-1} when $k_L = 10^{-10}$ ms^{-1} for the CCL. These hydraulic transmissivities correspond to "good contact" conditions as defined by Giroud (1997) in developing his semi-empirical equations (Rowe 1998).

The liner and foundation layer thickness were varied to test the influence of the different types of composite liners used in the USA, Canada, and France. The distance l' between the edges of the wrinkles at which there is an interaction between wrinkles is summarised in Tables 1 and 2 for CCLs and GCLs, respectively.

For CCLs, l' ranges between 2.5 and 8.7 m for h_w = 0.3 m and between 7.7 and 26.5 m for h_w = 3 m. For GCLs, l' ranges between 0.02 and 0.72 m for h_w = 0.3 m and between 0.05 and 1.86 m for h_w = 3 m. Taking into account the distance between parallel wrinkles, 5 to 10 m for HDPE geomembranes and 1 m for PVC geomembranes, the probability of encountering interacting wrinkles when the liner is a GCL is low, especially for HDPE geomembranes. Thus, the case of most interest for the study of the reduction rates of liquid flow due to two interacting wrinkles is that involving a CCL. This is illustrated by two examples in Section 5.4.

Table 1. Distance between edges of wrinkles at which interaction between wrinkles occurs for a geomembrane in contact with a CCL.

θ (m^2s^{-1})	k_L (ms^{-1})	k_f (ms^{-1})	H_L (m)	H_f (m)	h_w (m)	i_s	l'(m)
1.6×10^{-8}	10^{-9}		0.60	0	0.3	1.50	5.96
					3.0	6.00	15.35
1.6×10^{-8}	10^{-9}	10^{-7}	0.75	3	0.3	1.08	2.81
					3.0	1.80	8.43
1.6×10^{-8}	10^{-9}	10^{-6}	1.00	5	0.3	1.05	2.53
					3.0	1.50	7.72
1.6×10^{-8}	10^{-9}		5.00	0	0.3	1.06	6.17
					3.0	1.60	18.73
3.2×10^{-9}	10^{-10}		0.60	0	0.3	1.50	8.43
					3.0	6.00	21.72
3.2×10^{-9}	10^{-10}	10^{-7}	0.75	3	0.3	1.08	3.90
					3.0	1.80	11.71
3.2×10^{-9}	10^{-10}	10^{-6}	1.00	5	0.3	1.05	3.56
					3.0	1.50	10.89
3.2×10^{-9}	10^{-10}		5.00	0	0.3	1.06	8.72
					3.0	1.60	26.49

Table 2. Distance between the edge of wrinkles at which interaction between wrinkles occurs for a geomembrane in contact with a GCL.

θ (m^2s^{-1})	k_L (ms^{-1})	h_w (m)	i_s	l'(m)
6×10^{-12}	2×10^{-10}	0.3	1.49	0.03
		3.0	5.92	0.09
2×10^{-10}	2×10^{-10}	0.3	1.49	0.19
		3.0	5.92	0.50
10^{-10}	7×10^{-12}	0.3	1.49	0.72
		3.0	5.92	1.86
10^{-10}	2×10^{-10}	0.3	1.49	0.14
		3.0	5.92	0.35
10^{-10}	2×10^{-08}	0.3	1.49	0.02
		3.0	5.92	0.05

5.4 Reduction of Rate of Liquid Flow Due to Two Interacting Wrinkles

The following illustrates the potential reduction of the rate of liquid flow due to two interacting wrinkles in the case of a geomembrane in good contact with a CCL. Figure 8 shows the ratio Q/Q' as a function of the dimensionless quantity $\alpha l'$, where Q' is the rate of liquid flow obtained when the hydraulic head at MP is zero and there is no interaction under saturated flow conditions. Results are given for a range of hydraulic gradients in the soil below the wrinkle.

For a 0.6 m-thick CCL, with $k_L = 10^{-10}$ ms^{-1} and $\theta = 3.2 \times 10^{-9}$ m^{2}s^{-1}: $\alpha = 0.23$ m^{-1}. If $l' = 10$ m, then, $\alpha l' = 2.28$. Thus, if $h_w = 1$ m, $h_a = 0$, and $H_L = 0.6$ m, then, the gradient $i_s = 2.67$ and the ratio $Q/Q' = 0.88$ from Figure 8, implying a reduction in rate of liquid flow of 12% compared to the case of noninteracting wrinkles. Q can then be calculated after calculating Q' from Equation 49.

As a second example, consider a 5 m-thick CCL, with $k_L = 10^{-9}$ ms^{-1} and $\theta = 1.6 \times 10^{-8}$ m^{2}s^{-1}:

$$\alpha = \sqrt{\frac{k_s}{\theta H_L}} = \sqrt{\frac{10^{-9}}{(1.6 \times 10^{-8})(5)}} = 0.112 \ \text{m}^{-1}$$

Taking $l' = 5$ m, then $\alpha l' = 0.56$. For $h_w = 0.3$ m, the maximum gradient $i_s = 1.06$, and one can obtain $Q/Q' = 0.82$. Thus, the reduction of rate of liquid flow is 18% when compared to the case of noninteracting wrinkles.

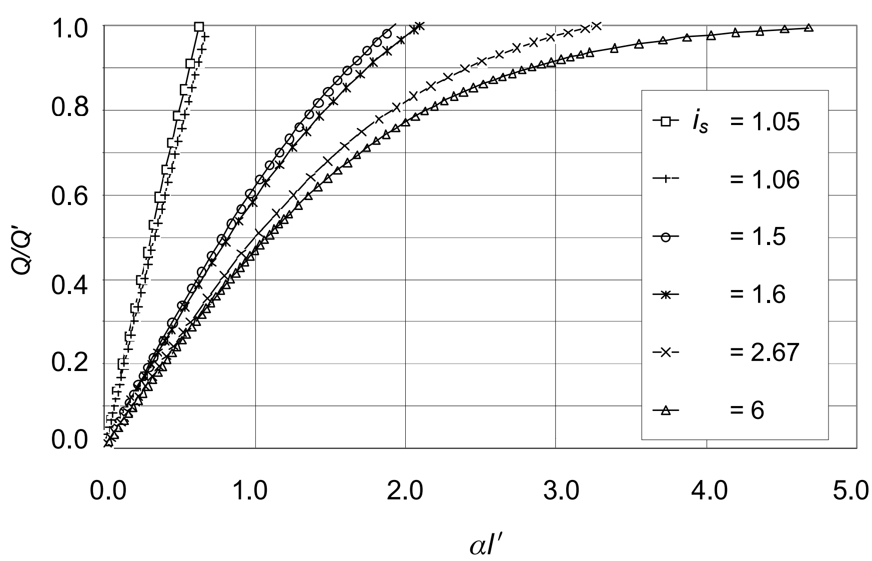

Figure 8. Effect of interaction between wrinkles on flow rate as a function of $\alpha l'$ for various hydraulic gradient values.

6 CONCLUSIONS

A general framework for calculating the rate of liquid flow through a composite liner with holes has been presented. The solutions given for a circular hole in contact with the soil liner and a wrinkled geomembrane (with a hole in the wrinkle) could be used for interpreting data from laboratory tests, modelling expected field conditions, and/or interpreting field leakage data. It is shown that a number of existing solutions arise from the general solution as special cases. Finally, it is shown that interaction between wrinkles can be readily considered. This may be of some importance for geomembranes on compacted clay, but is far less likely to be of importance for geomembranes over GCLs based on typical published data for the hydraulic transmissivity of transmissive layers.

ACKNOWLEDGEMENTS

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REFERENCES

Brown, K.W., Thomas, J.C., Lytton, R.L., Jayawickrama, P. and Bhart, S., 1987, "*Quantification of Leakage Rates through Holes in Landfill Liners*", US EPA Report CR810940, Cincinnati, Ohio, USA, 147 p.

Fukuoka, M., 1986, "Large scale permeability test for geomembrane subgrade system", *Proceedings of the Third International Conference on Geotextiles*, Vol. 3, Vienna, Austria, April 1986, pp. 917-922.

Giroud, J.P., 1997, "Equations for Calculating the Rate of Liquid Migration Through Composite Liners Due to Geomembrane Defects", *Geosynthetics International*, Vol. 4, Nos. 3-4, pp. 335-348.

Giroud, J.P. and Bonaparte, R., 1989, "Leakage through Liners Constructed with Geomembranes - Part II. Composite Liners", *Geotextiles and Geomembranes*, Vol. 8, No. 2, pp. 71-111.

Giroud, J.P., Khire, M.V. and Soderman, K.L., 1997, "Liquid Migration Through Defects in a Geomembrane Overlain and Underlain by Permeable Media", *Geosynthetics International*, Vol. 4, Nos. 3-4, pp. 293-321.

Giroud, J.P. and Morel, F., 1992, "Analysis of Geomembrane Wrinkles", *Geotextiles and Geomembranes*, Vol. 11, No. 3, pp. 255-276 (Erratum: 1993, Vol. 12, No. 4, p. 378).

Giroud, J.P., Soderman, K.L., Khire, M.V. and Badu-Tweneboah, K., 1998, "New Developments in Landfill Liner Leakage Evaluation", *Proceedings of the Sixth International Conference on Geosynthetics*, IFAI, Vol. 1, Atlanta, Georgia, USA, March 1998, pp. 261-268.

Harpur, W.A., Wilson-Fahmy, R.F. and Koerner, R.M., 1994, "Evaluation of the Contact between Geosynthetic Clay Liners and Geomembranes in Terms of Transmissivity", *Geosynthetic Liner Systems: Innovations, Concerns and Designs*, Koerner, R.M. and Wilson-Fahmy, R.F., Editors, IFAI, 1994, Proceedings of the Seventh Symposium of the Geosynthetic Research Institute, Philadelphia, Pennsylvania, USA, December 1993, pp. 143-154.

Jayawickrama, P.W., Brown, K.W., Thomas, J.C. and Lytton, R.L., 1988, "Leakage rates through flaws in membrane liners", *Journal of Environmental Engineering*, Vol. 114, No. 6, 1401-1420.

Liu, Z.Y., 1998, "Scheme of Using Geosynthetics to Treat Cracks on a Reservoir Blanket", *Proceedings of the Sixth International Conference on Geosynthetics*, IFAI, Vol. 2, Atlanta, Georgia, USA, March 1998, pp. 1121-1124.

Pelte, T., Pierson, P. and Gourc, J.P., 1994, "Thermal Analysis of Geomembranes Exposed to Solar Radiation", *Geosynthetics International*, Vol. 1, No. 1, pp. 21-44.

Rollin, A.L. and Jacquelin, T., 2000, "Geomembrane Failures: Lessons Learned from Geo-Electrical Leaks Surveys", to be published in *Lessons Learned from Failures Associated with Geosynthetics*, J.P. Giroud, Editor.

Rowe, R.K., 1998, "Geosynthetics and the minimization of contaminant migration through barrier systems beneath solid waste", Keynote paper, *Proceedings of the Sixth International Conference on Geosynthetics*, IFAI, Vol. 1, Atlanta, Georgia, USA, March 1998, pp. 27-103.

Touze-Foltz, N., 1999, "Large Scale Tests for the Evaluation of Composite Liners Hydraulic Performance: a Preliminary Study", *Proceedings of the Seventh International Waste Management and Landfill Symposium*, Vol. 3, S. Margherita di Pula, Cagliari, Sardinia, Italy, October 1999, pp. 157-164.

NOTATIONS

Basic SI units are given in parentheses.

A, A_p , A_Q ,	=	constants (dimensionless)
a	=	area of hole in geomembrane (m^2)
B, B_p , B_Q	=	constants (dimensionless)
b	=	half width of wrinkle (m)
b_1	=	half width of Wrinkle 1 for two interacting wrinkles (m)
b_2	=	half width of Wrinkle 2 for two interacting wrinkles (m)
C	=	$H_L + H_f - h_a$ (m)
E	=	coefficient, value dependent on boundary conditions (Equation 28) (m)
F	=	coefficient, value dependent on boundary conditions (Equation 28) (m)
g	=	gravitational acceleration (ms^{-2})
H_f	=	thickness of foundation layer (m)

H_L	=	thickness of soil liner (CCL or GCL) (m)
h	=	hydraulic head in transmissive layer (m)
h_a	=	potentiometric head in aquifer or at bottom of foundation layer (m)
h_s	=	specified hydraulic head in transmissive layer at $r = R_c$ (m)
h_w	=	leachate head acting on top of geomembrane (m)
h_1	=	hydraulic head for x between b_1 and l_1 (m)
h_2	=	hydraulic head for x between l_1 and $l_{bw} - b_2$ (m)
I_0	=	modified Bessel function of zero order (dimensionless)
I_1	=	modified Bessel function of first order (dimensionless)
i_s	=	maximum mean gradient across soil liner and foundation layer (dimensionless)
K_0	=	modified Bessel function of zero order (dimensionless)
K_1	=	modified Bessel function of first order (dimensionless)
k_f	=	hydraulic conductivity of foundation layer (ms^{-1})
k_L	=	hydraulic conductivity of soil liner (GCL or CCL) (ms^{-1})
k_{OM}	=	hydraulic conductivity of leachate collection system (ms^{-1})
k_s	=	harmonic mean hydraulic conductivity of soil liner and foundation layer (ms^{-1})
L	=	length of wrinkle (m)
l_{bw}	=	$l_1 + l_2$ (m)
l'	=	$l_{bw} - b_1 - b_2$ (m)
l_1	=	width of Wrinkle 1 in two interacting wrinkles case (m)
l_2	=	width of Wrinkle 2 in two interacting wrinkles case (m)
Q	=	rate of liquid flow through hole in geomembrane (m^3s^{-1})
Q'	=	rate of liquid flow from zone between edges of wrinkles (m^3s^{-1})
Q_r	=	radial rate of liquid flow in transmissive layer for circular problem (m^3s^{-1})
Q_s	=	rate of liquid flow into soil (soil liner + foundation layer) (m^3s^{-1})
Q_x	=	rate of liquid flow in transmissive layer for two-dimensional problem (m^3s^{-1})
q_i	=	rate of percolation through waste reaching leachate collection layer per unit area (ms^{-1})
R_c	=	physical radius of system studied in axi-symmetric case (m)
R_w	=	limit radius of wetted area for validity of axi-symmetric solutions (m)
r	=	radial distance (m)
r_0	=	radius of hole in geomembrane (m)
X_c	=	width of cell or system studied in damaged wrinkle case (m)
X_w	=	limit width for validity of two-dimensional solutions (m)
x	=	horizontal distance (m)

z	=	vertical distance (m)
z_{hole}	=	vertical position of center of hole (m)
α	=	$\{k_s/[(H_L + H_f)\,\theta]\}^{0.5}$ (m^{-1})
θ	=	hydraulic transmissivity of transmissive layer (m^{2}s^{-1})

ABBREVIATIONS

CCL:	compacted clay liner
GCL:	geosynthetic clay liner
HDPE:	high density polyethylene
MP:	meeting point of hydraulic head profiles for two interacting wrinkles
PVC:	polyvinyl chloride

Erratum

LIQUID FLOW THROUGH COMPOSITE LINERS DUE TO GEOMEMBRANE DEFECTS: ANALYTICAL SOLUTIONS FOR AXI-SYMMETRIC AND TWO-DIMENSIONAL PROBLEMS

TECHNICAL PAPER FOR ERRATUM: Touze-Foltz, N., Rowe, R.K. and Duquennoi, C., 1999, "Liquid Flow Through Composite Liners due to Geomembrane Defects: Analytical Solutions for Axi-Symmetric and Two-Dimensional Problems", *Geosynthetics International*, Vol. 6, No. 6, pp. 455-479.

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The authors inadvertently included a minus sign in Equation 8, p. 460 in their technical paper, which appeared in *Geosynthetics International*, Vol. 6, No. 6.

ERRATUM FOR SECTION: 3.1.1 *Governing Equation*

On p. 460, Equation 8 should read :

$$\mathrm{d}Q_s(r) = 2\pi r k_s \left(1 + \frac{h - h_a}{H_f + H_L}\right) \mathrm{d}r \tag{8}$$